Handout for Week 10: *Intrinsic* Metavocabularies for Reason Relations

Philosophy of Language.

**Metavocabularies of Reason**:

Pragmatics, Semantics, and Logic

https://sites.pitt.edu/~rbrandom/Courses

Plan:

1. Bimodal conceptual realism and Ulf's isomorphism.
2. Implication-space semantics and Bob's isomorphism.
3. Truth-value model theory vs. Inferential Entailment Roles

1) Conceptual realism:

**McDowell**: "The conceptual has no outer boundary."
**Wittgenstein**: "When we say, and mean, that such-and-such is the case, we—and our meaning—do not stop anywhere short of the fact; but we mean: this—is—so."
***Tractatus***: "The world is everything that is the case. It is the totality of facts, not of things."
**Frege**: "A fact is a thought that is true."
To be in conceptual shape or to have conceptual content is to stand in reason relations of consequence and incompatibility to other such items.

Bimodal conceptual realism:

Relations of consequence and incompatibility can be specified in two sorts of vocabulary:

- Deontic normative vocabulary, on the *subjective* side of discursive activity, and
- Alethic modal vocabulary, on the *objective* side of how things are.

Because these reason relations can come in two flavors, deontic and alethic, thoughts and facts are both intelligible as conceptually articulated.

Bimodal *hylomorphic* conceptual realism:

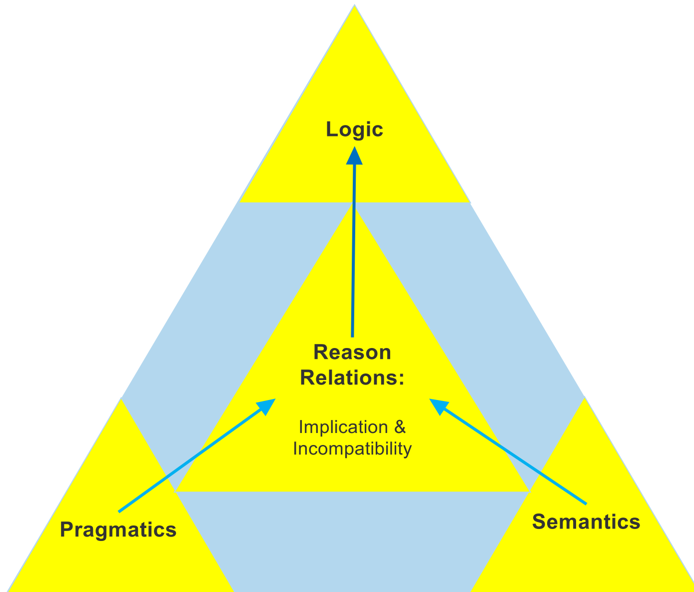
Two versions of what is in conceptual shape and shows up in two guises,:

a) One *content* specified in two *metavocabularies*: normative pragmatic and alethic semantic. (Bob's Hegel.)
b) One *form* for two *matters*: mind and world. (Ulf's Aristotle.)

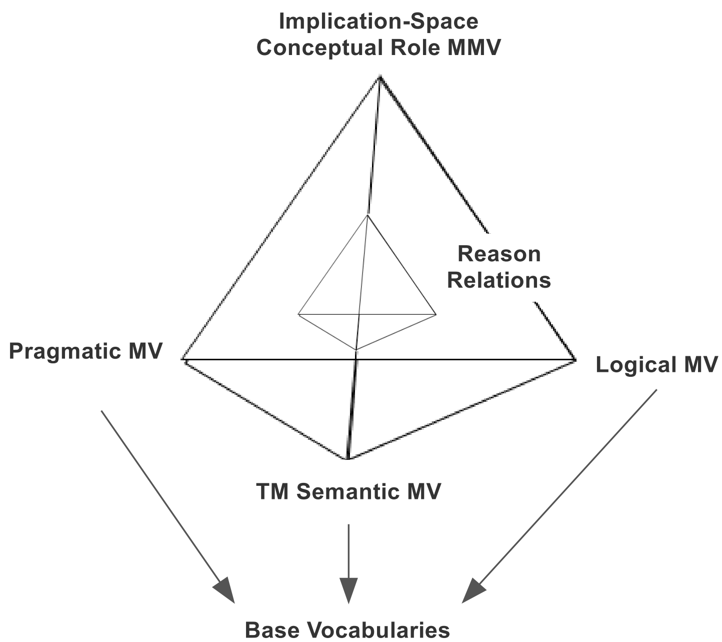
2) Implication-space conceptual role (meta-)metavocabulary:

Like *logical* vocabulary, the implication-space conceptual role rational metavocabulary is *also* universal and comprehensive, able to codify the reason relations of arbitrary base vocabularies and metavocabularies.
Slogan: "**One rational metavocabulary to rule them all!**"

The advance is from this picture:




to this one:




(Pyramid power.)

<u>Implication-space conceptual role semantics is the *intrinsic* semantics of reason relations</u>:

The *intrinsicness* ("intrinsicality"?) of the semantics consists in its needing *nothing* else in addition to the base vocabulary to determine the whole semantics:

i) The *universe* is the set $\mathscr{P}$(L)x$\mathscr{P}$(L), thought of as *candidate implications* <Γ,Δ>. It is determined entirely by the lexicon L of the base vocabulary.

ii) The *mereological* element of *structure* on that universe is the *commutative monoid* of *adjunction*, which is wholly definable set-theoretically from the structure of the elements of the implication space. $X \uplus Y = Z$, where $X = <X_1, X_2>$, $Y = <Y_1, Y_2>$ and $Z = <Z_1, Z_2>$ iff $X_1 \cup Y_1 = Z_1$ and $X_2 \cup Y_2 = Z_2$.

iii) Further *modal* structure on the universe is the *distinguished subset* $\mathbf{I} \subseteq \mathscr{P}(L) x \mathscr{P}(L)$ of elements $<\Gamma, \Delta>$ where $\Gamma |\sim \Delta$, the *good implications* (including incoherent sets, so incompatibilities) of the base vocabulary.

There might be constraints on **I**, such as that all candidate implications of the form $<\Gamma \cup \{A\}, \Delta \cup \{A\}>$ are elements of **I**. (That is CO.) But this is determined wholly by the base vocabulary.

iv) The *space of semantic interpretants* of sentences and sets of sentences (to be assigned by the *v* function in (v)) is then the set of all sets of pairs of sets of sentences: $S = \mathscr{P}(\mathscr{P}(L) x \mathscr{P}(L))$.

v) The *interpretation function v* assigns $<X,Y> \in v(<\Gamma, \Delta>$ iff $<\Gamma \cup X, \Delta \cup Y> \in \mathbf{I}$.

vi) In terms of these semantic interpretations of (candidate) *implications*, we can then assign *inferential roles* to individual sentences. Each sentence is assigned the ordered pair of (the *v*-closures of) the *v*-set of $<A, \varnothing>$, A's *premissory role*, and the *v*-set of $<\varnothing, A>$, A's *conclusory* role.

vii) We can now define not only reason relations of implication and incompatibility for the original vocabulary, but also for the logically extended vocabulary definable (elaborated) from that base vocabulary.

(Semantic Entailment). We say that A semantically entails B relative to a model M if the closure of the combination of A (as premise) and B (as conclusion) consists of only good implications:

$$A \models_M B \text{ iff } (([A]_P)^v \uplus ([B]_C)^v)^{vv} \subseteq \mathbf{I}_M.$$

The closure of the adjunction of A as a premise with B as a conclusion consists only of good implications.

<u>Comparison with truthmaker semantics (*extrinsic* because relying on metaphysics)</u>:

i) The *universe* is a set of *states*, that must be **stipulated**, in some *metaphysical* vocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

ii) The *mereological* element of *structure* on that universe is a *commutative monoid* of *fusion*, which must also be **stipulated**, in some *metaphysical* metavocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

iii) Further *modal* structure is provided by a distinguished subset of the universe, the *possible* states, which must also be **stipulated**, in some *metaphysical* metavocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

iv) The *space of semantic intepretants* of sentences (and sets of sentences), to be assigned to sentences by the interpretation function in (v), then is the set of pairs of sets of states. This can be *defined* from (i) and (ii).

v) The *interpretation function* that assigns each sentence (or set of sentences) a pair of sets of states, as its (exact) *truthmakers* and (exact) *falsemakers*. This must be **stipulated**, in some semantic metavocabulary quite distinct from the base vocabulary for which a semantics is to be provided.

vi) Various different *reason relations* among sentences can then be defined, appealing either *just* to the mereological structure of the universe and semantic space (as Fine does for consequence and incompatibility—which will appeal to false-makers), or also to the *modal* structure, as Ulf's definition of consequence does (and a stronger notion of <u>incompatibility</u> that appealed to *impossibility* would).

Slogan: **Implication-space conceptual role semantics is *semantics without metaphysics*.**

<u>Implication-space semantics and truthmaker semantics share a structure:</u>
Both have commutative monoids plus partition of the space of the monoid.
As a result:
*If* reason relations of implication and incompatibility are defined in the truthmaker framework in a particular way—what we claim is the *right* way—then **exactly the same relations of implication and incompatibility are specifiable in the implication-space conceptual role framework as in the truthmaker framework**.

For one direction: Beginning with a truth-maker model, one can define an implicational phase space that corresponds to it in the sense of defining exactly the same implications and incompatibilities. We are given a truth-maker model of a language $L_0$, defined on a modalized state space $\langle S,S^{\Diamond},\sqcup\rangle$, which assigns to each sentence $A\in L_0$ a pair of sets of states $\langle v(A),f(A)\rangle$ understood as verifiers and falsifiers of that sentence. The points of the implicational phase space being defined are ordered pairs of sets of sentences of $L_0$. These are the candidate implications. What corresponds to fusion, $\sqcup$, is adjunction: $\langle\Gamma,\Delta\rangle \uplus \langle\Theta,\Psi\rangle = \langle\Gamma\cup\Theta,\Delta\cup\Psi\rangle$, as usually defined in implicational phase space semantics. It remains to compute $\mathbf{I}_0$, the set of *good* implications. We do that using the consequence relation Hlobil defined to mimic the Restall-Ripley bilateral understanding of the multisuccedent turnstile:

$$\langle\Gamma,\Delta\rangle\in\mathbf{I}_0 \quad \text{iff} \quad \forall s,t\in S[\ (\forall G\in\Gamma[s\in v(G)]\ \&\ \forall D\in\Delta\ [t\in f(D)]\ ) \Rightarrow s\sqcup t\notin S^{\Diamond}\ ].$$

That is, $\langle\Gamma,\Delta\rangle$ is a good implication just in case the fusion of any state s that verifies all of $\Gamma$ and any state t that falsifies all of $\Delta$ is an impossible state, in the truth-maker model. This construction obviously guarantees that exactly the same implications will hold in the implicational phase space, that is, be elements of $\mathbf{I}_0$, as satisfy the Hlobil consequence relation in the truth-maker model.

As for incompatibilities, in the truth-maker setting, two *states* s and t are incompatible just in case their fusion is an impossible state. Two *sentences* A and B are incompatible just in case any fusion of a verifier of the one with a verifier of the other is an impossible state. More generally, a set $\Gamma$ of sentences is *incoherent* in case any fusion of verifiers of all its elements is an *impossible* state. Given the definition of the set of good implications $\mathbf{I}_0$ just offered, this is equivalent to $<\Gamma,\varnothing>\in\mathbf{I}_0$. The incompatibilities are represented in the implicational phase space semantics just by good implications with empty right-hand sides.

**So there is a straightforward method for taking any truth-maker model defined on a modalized state space and defining from it an implicational phase space model that has exactly the same reason relations of implication and incompatibility**.

For the other direction: **Beginning with an implicational phase space, one can define a truth-maker model (an interpreted modalized state space) that corresponds to it in the sense of defining exactly the same implications and incompatibilities**.
We are given an implicational phase space defined on a language $L_0$, $<\mathcal{P}(L_0) \times \mathcal{P}(L_0), \mathbf{I}_0>$. The states will be candidate implications. $S = \mathcal{P}(L_0) \times \mathcal{P}(L_0)$. $\sqcup$ is adjunction: $<\Gamma,\Delta> \sqcup <\Theta,\Psi> = <\Gamma\cup\Theta,\Delta\cup\Psi>$. In the Hlobil truth-maker definition of consequence, the *good* implications correspond to *im*possible states. So the subset of *possible* states is defined by $S^{\Diamond} = S\text{-}\mathbf{I}_0$. It remains to define the model function m, which assigns to each $A\in L_0$ a pair of subsets of S, $<v(A),f(A)>$, where $v(A)\subseteq L_0$ and $f(A)\subseteq L_0$, such that:

$$<\Gamma,\Delta>\in\mathbf{I}_0 \text{ iff } \forall s,t\in S[(\Gamma=\{G_1\ldots G_n\} \ \&\ g_1\in v(G_1) \ \&\ldots g_n\in v(G_n) \ \&\ s=g_1\sqcup\ldots\sqcup g_n \ \&$$
$$\Delta=\{D_1\ldots D_n\} \ \&\ d_1\in v(D_1) \ \&\ldots d_n\in v(D_n) \ \&\ t=d_1\sqcup\ldots\sqcup d_n ) \Rightarrow s\sqcup t\notin S^{\Diamond} ].$$

For various metatheoretic purposes, Fine employs "canonical" truth-making models, in which the verifier of a (logically atomic) sentence is just that sentence and the falsifier of that sentence is just the negation of that sentence. (His requirement that the fusion of any verifiers of A will be a verifier of A and the fusion of any falsifiers of A will also be a falsifier of A is then trivially satisfied, since there is only one.) We can combine that idea with Kaplan's standard representation of the proposition expressed by A as the pair $< <A, \varnothing>, <\varnothing, A> >$, and do without the formation of falsifying literals by appeal to negation by defining the verifiers of A by $v(A) = <A, \varnothing>$ and the falsifiers of A by $f(A) = <\varnothing, A>$. We want to implement Hlobil's definition of implication (generalizing C. I. Lewis's strict implication to Fine's truthmaker semantic framework), that an implication $\Gamma|\!\sim\Delta$ is good in the truth-maker setting just in case the fusion of any verifier of all of $\Gamma$ and any falsifier of all of $\Delta$ is an impossible state. To do that, we need to say what it is for a state (defined in the implicational phase space, that is, a candidate implication) to "verify all of $\Gamma$" and to "falsify all of $\Delta$." We can extend the single-sentence definitions as follows. If $\Gamma=\{G_1\ldots G_n\}$ and $\Delta=\{D_1\ldots D_m\}$:

$$v(\Gamma) = <\Gamma,\varnothing> = <G_1,\varnothing>\uplus\ldots\uplus <G_n,\varnothing>.$$

$$f(\Delta) = \langle\varnothing,\Delta\rangle = \langle\varnothing,D_1\rangle \uplus \ldots \uplus \langle\varnothing,D_m\rangle.$$

That is, the implication (standing in for a state) $\langle\Gamma,\varnothing\rangle$ counts as verifying all of $\Gamma$ because it is the adjunction of the verifiers of each element of $\Gamma$. (In this "canonical" modalized state-space model, sets of sentences, like individual sentences, only have single states=implications as verifiers.) And similarly for falsifiers.

To show that this works, in the sense of yielding the same implications in the truth-maker model that are good in the original implicational phase space, we must show that

$$\langle\Gamma,\Delta\rangle\in\mathbf{I}_0 \text{ iff } \forall s,t\in S[\,(\forall G\in\Gamma[s\in v(G)]\ \&\ \forall D\in\Delta\,[t\in f(D)]\,) \Rightarrow s\sqcup t\notin S^{\Diamond}\,].$$

<u>To show the left-to-right direction $\Rightarrow$</u>: If $\langle\Gamma,\Delta\rangle\in\mathbf{I}_0$ then $v(\Gamma)=\langle\Gamma,\varnothing\rangle$ and $f(\Delta)=\langle\varnothing,\Delta\rangle$. So $v(\Gamma)\sqcup f(\Delta)=\langle\Gamma,\Delta\rangle$. Since by hypothesis $\langle\Gamma,\Delta\rangle\in\mathbf{I}_0$, by the definition of $S^{\Diamond}$ as $S\text{-}\mathbf{I}_0$, it follows that $\langle\Gamma,\Delta\rangle\notin S^{\Diamond}$, that is, that the state $\langle\Gamma,\Delta\rangle$ is an impossible state. It is the fusion of *the* verifier of $\Gamma$, $\langle\Gamma,\varnothing\rangle$ and *the* falsifier of $\Delta$ $\langle\varnothing,\Delta\rangle$ because it is the result of adjoining them.

<u>To show the right-to-left direction $\Leftarrow$</u>: If $\forall s,t\in S[(\Gamma=\{G_1\ldots G_n\}\ \&\ g_1\in v(G_1)\ \&\ldots g_n\in v(G_n)\ \&\ s=g_1\sqcup\ldots\sqcup g_n\ \&\ \Delta=\{D_1\ldots D_n\}\ \&\ d_1\in v(D_1)\ \&\ldots d_n\in v(D_n)\ \&\ t=d_1\sqcup\ldots\sqcup d_n\,) \Rightarrow s\sqcup t\notin S^{\Diamond}\,]$, then $s = v(\Gamma)$ and $t = f(\Delta)$, so $v(\Gamma)\sqcup f(\Delta)=\langle\Gamma,\Delta\rangle \notin S^{\Diamond}$. Since $S^{\Diamond} = S\text{-}\mathbf{I}_0$ and $\langle\Gamma,\Delta\rangle\in S$, $\langle\Gamma,\Delta\rangle\in\mathbf{I}_0$.

As for incompatibility, we must show that A and B are truth-maker incompatible ($\Gamma$is truth-maker incoherent), that is, $\forall s,t\in S[s\in v(A)\ \&\ t\in v(B) \Rightarrow s\sqcup t\notin S^{\Diamond}]$, (or more generally, $v(\Gamma)\notin S^{\Diamond}$) iff $\langle\{A,B\},\varnothing\rangle\in\mathbf{I}_0$ (or more generally, $\langle\Gamma,\varnothing\rangle\in\mathbf{I}_0$).

<u>To show the left-to-right direction $\Rightarrow$</u>: If $\forall s,t\in S[s\in v(A)\ \&\ t\in v(B) \Rightarrow s\sqcup t\notin S^{\Diamond}]$, then since $v(A) = \langle A,\varnothing\rangle$ and $v(B) = \langle B,\varnothing\rangle$, and since $\sqcup$ is adjunction, $s\sqcup t = \langle\{A\}\cup\{B\},\varnothing\rangle = \langle\{A,B\},\varnothing\rangle$. Since $\Rightarrow s\sqcup t\notin S^{\Diamond}$, $s\sqcup t = \langle\{A,B\},\varnothing\rangle\in\mathbf{I}_0$. This works for arbitrary iterations of $\sqcup$, which gives the more general $\Gamma$case.

<u>To show the right-to-left direction $\Leftarrow$</u>: If $\langle\{A,B\},\varnothing\rangle\in\mathbf{I}_0$, then $\langle\{A\}\cup\{B\},\varnothing\rangle\in\mathbf{I}_0$.
Since $\sqcup$ is adjunction, $\langle A,\varnothing\rangle\sqcup\langle B,\varnothing\rangle\in\mathbf{I}_0$. But $v(A) = \langle A,\varnothing\rangle$ and $v(B) =\langle B,\varnothing\rangle$.
So $v(A)\sqcup v(B)\in\mathbf{I}_0$. Since $S^{\Diamond} = S\text{-}\mathbf{I}_0$, $v(A)\sqcup v(B)\notin S^{\Diamond}$. That is truth-maker incompatibility of A and B. This works for arbitrary iterations of $\sqcup$, which gives the more general $\Gamma$ case.
QED.

3) <u>Comparing the inferential entailment role rational metavocabulary with multivalued truth-value logical semantics</u>:

| $\neg$ | **A** | **$\neg$A** |
|---|---|---|
| | T = 1 | F = 0 |
| | U = ½ | U = ½ |
| | F = 0 | T = 1 |

| **A&B B:** **A** | T = 1 | U = ½ | F = 0 |
|---|---|---|---|
| T = 1 | T = 1 | U = ½ | F = 0 |
| U = ½ | U = ½ | U = ½ | F = 0 |
| F = 0 | F = 0 | F = 0 | F = 0 |

| **A∨B B:** **A** | T = 1 | U = ½ | F = 0 |
|---|---|---|---|
| T = 1 | T = 1 | T = 1 | T = 1 |
| U = ½ | T = 1 | U = ½ | U = ½ |
| F = 0 | T = 1 | U = ½ | F = 0 |

K3 (Weak Kleene) Definition of Consequence:

| **A\|~B B:** **A** | T = 1 | U = ½ | F = 0 |
|---|---|---|---|
| T = 1 * | ✓ | X | X |
| U = ½ | ✓ | ✓ | ✓ |
| F = 0 | ✓ | ✓ | ✓ |

LP (Graham Priest's "Logic of Paradox") Definition of Consequence:

| **A\|~B B:** **A** | T = 1 | U = ½ | F = 0 |
|---|---|---|---|
| T = 1 * | ✓ | ✓ | X |
| U = ½ * | ✓ | ✓ | X |
| F = 0 | ✓ | ✓ | ✓ |

K3 is the logic of pure *premissory* roles (the roles sentences play as premises of implications)
and
LP is the logic of pure *conclusory* roles (the roles sentences play as conclusions of implications).

<u>General Definition of Inferential Entailments</u>:

$$A^P, B^C \Rightarrow C^P, D^C$$

iff

$$[A]_P \cap [B]_C \subseteq (([C]_P)^{\nu} \cup ([D]_C)^{\nu})^{\nu}.$$

This formulation codifies the case where whenever A occurs as a premise and B as a conclusion, substituting C for A as a premise and D for B as a conclusion will never turn a good implication into a bad one.

This *mixes* premissory and conclusory roles, and specifies a much more complex relation among sentences than simple premissory and conclusory roles does.

It accordingly provides a much finer expressive scalpel for exposing and dissecting the fine structure of relations among the conceptual roles played by sentences.